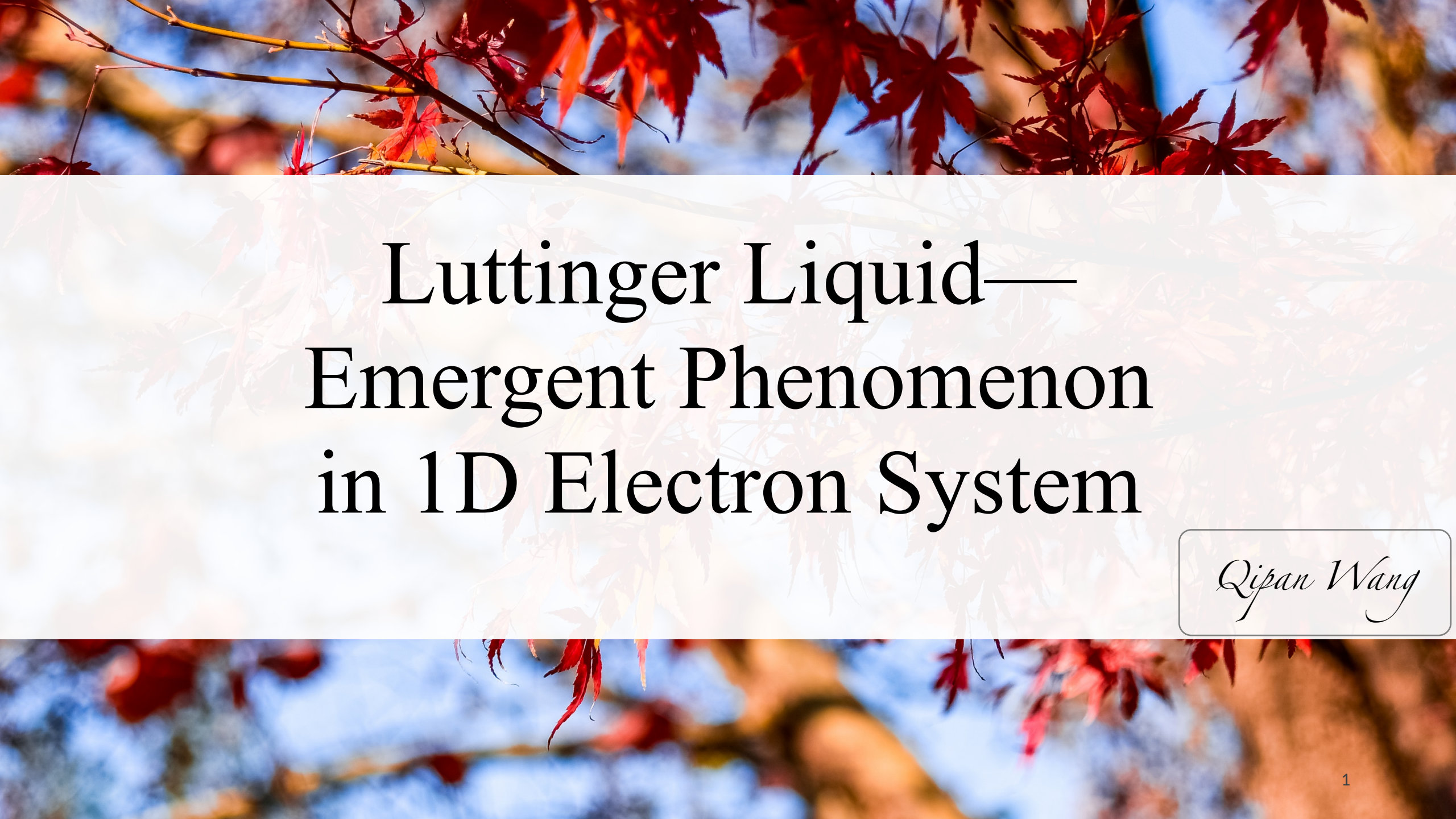

Luttinger Liquid— Emergent Phenomenon in 1D Electron System

Qipan Wang

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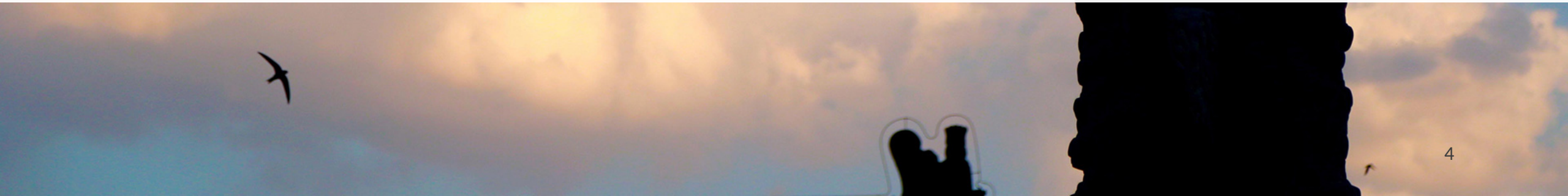
01 Part One

Landau Fermi Liquid: Basic Idea and Failure



Landau Fermi Liquid

Free Electron Gas model

Basic Idea of Drude Model:

Only consider kinetic energy: $H = \sum_i p_i^2/2m$, **ignoring all interactions (except instantaneous collisions)**

Electrons were treated as independent classical particles

Electrons follow Maxwell-Boltzmann distribution

Successes:

Explanation of electrical conductivity (ac conductivity of metals) and thermal conductivity

Quantitative explanation of the Wiedemann-Franz law

Further Development: Sommerfeld Model

Electrons are plane waves traveling in free space

Electrons follow Fermi-Dirac Distribution

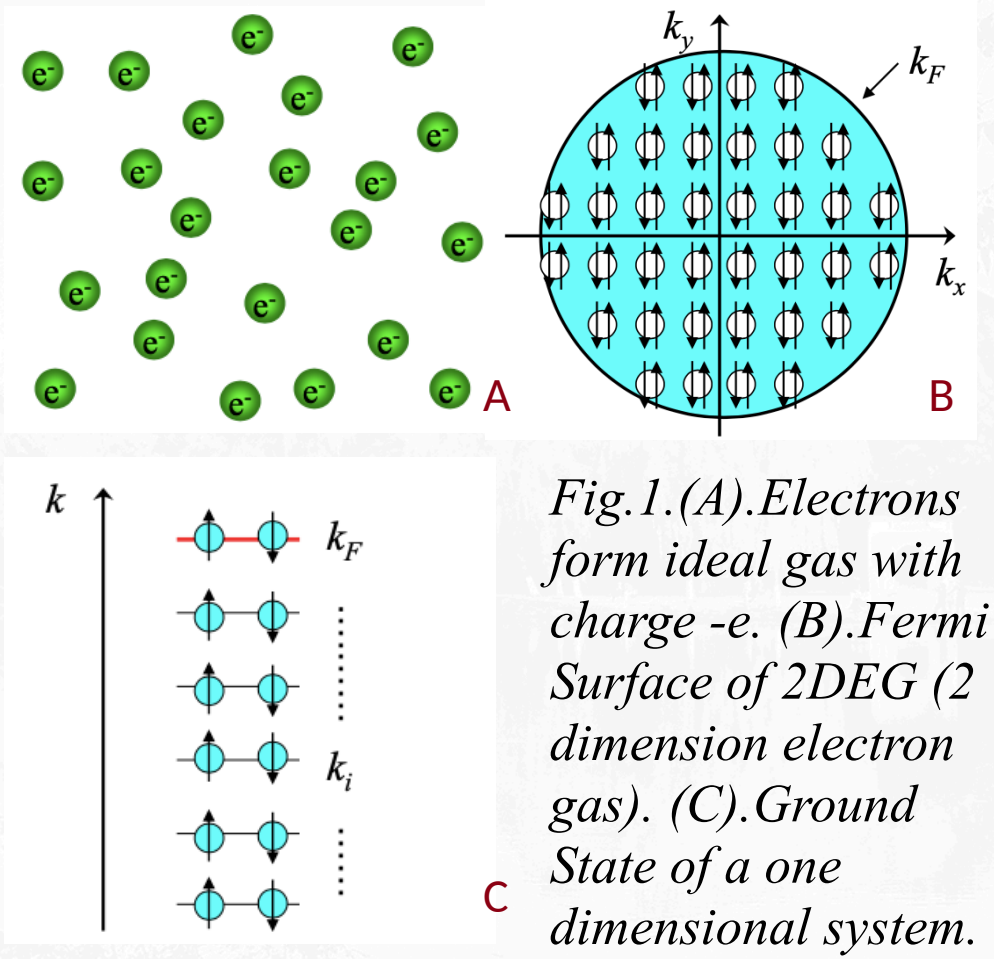


Fig.1.(A).Electrons form ideal gas with charge -e. (B).Fermi Surface of 2DEG (2 dimension electron gas). (C).Ground State of a one dimensional system.

Landau Fermi Liquid

Motivation of Landau Fermi Liquid

Physical properties of metals suggest that electrons behave like independent particles, even though the Coulomb interaction in most metals are much larger than the electron kinetic energy.

If the interaction between electrons is not too strong, then a **non-interacting quasi-particle** (qp, named by Landau, **has the same charge, spin and momentum as non-interacting electron**) picture is still a good first approximation.

Its behavior is qualitatively the same as non-interacting electrons. Energy levels differ from non-interacting electrons by interactions, with an effective mass $m^* \neq m_e$ introduced, causing quantitative change in some properties. Low energy excitations can be accounted for by the Landau interaction function f.

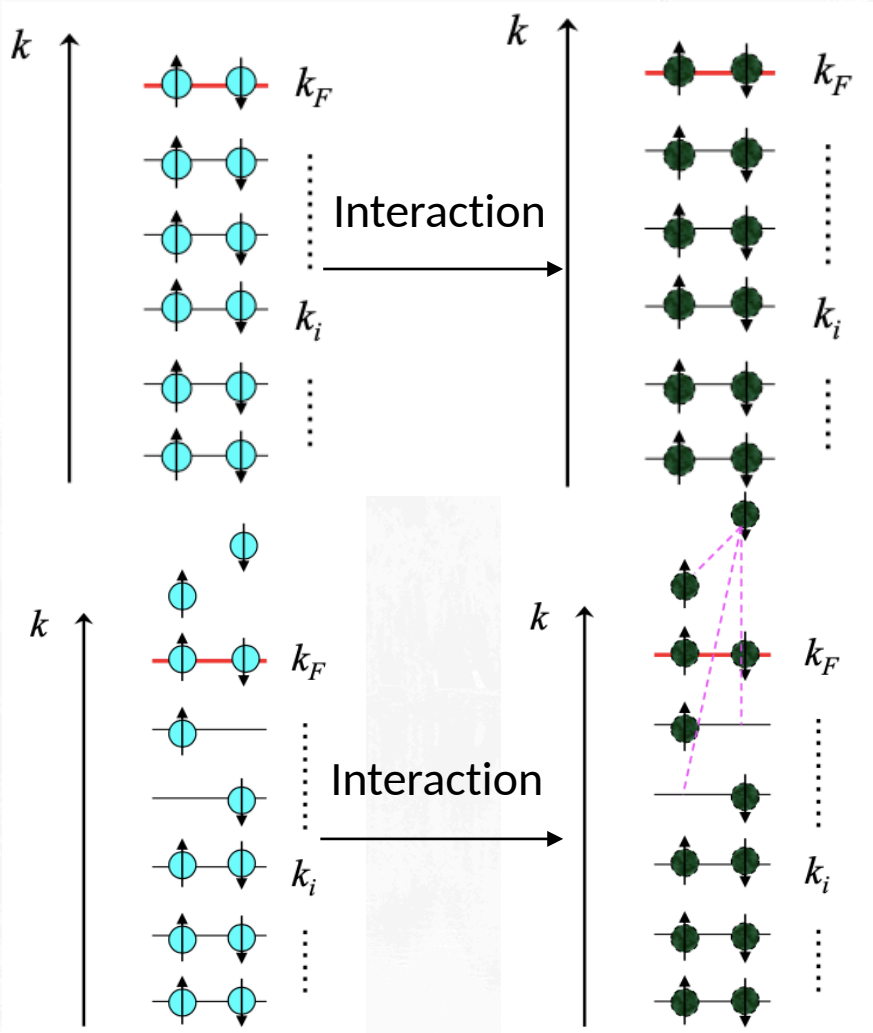


Fig.2.(Up).Non-interacting electron gas (Left) to qps (Right). (Down).Low energy excitation of free electrons and qps.

Adiabatic continuity (proposed by Anderson in 1981):

Labels associated with eigenstates are more robust against perturbations than the eigenstates themselves.

A low lying excited state of an interacting Fermion system can be constructed by:

1. preparing a low lying excited state of non-interacting ideal Fermi-liquid (e.g., adding an electron/hole above/below the Fermi level);
2. switching on the interaction *suitably slowly*: it should be switched on before the state is totally damped. It is possible because an electron/hole near the Fermi surface damps very slowly.

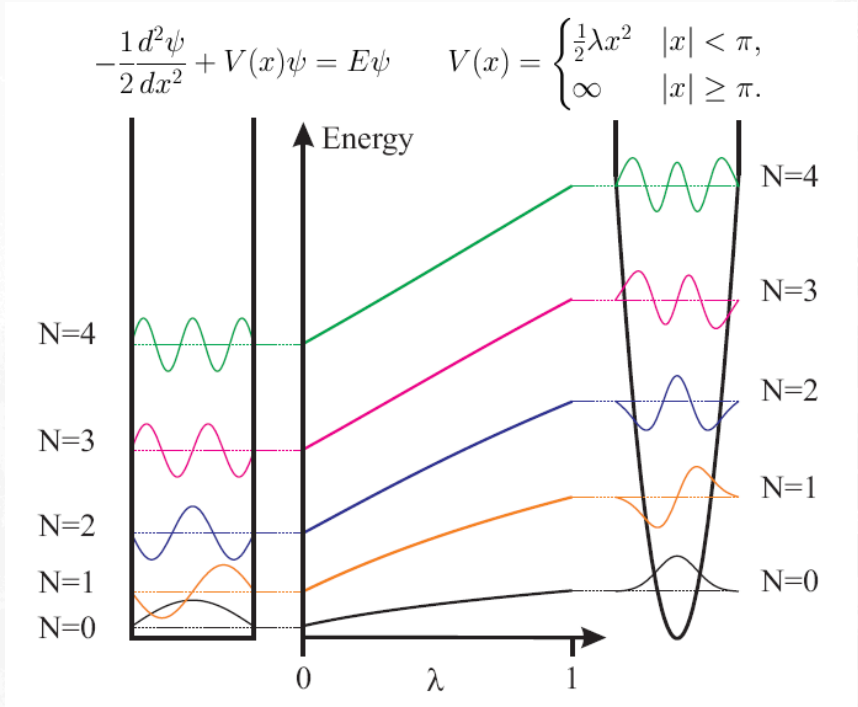


Fig.3. A simple illustration of adiabatic continuity. By slowly turning on a weak quadratic potential, the new eigenstates change but the number of nodes still remains a good label of the new eigenstates.

Landau Fermi Liquid

Failure in 1D system

In 1D, Fermi surface is two dots: $\pm k_F$. For q $\rightarrow$ 0, the range of allowed low-energy excitations shrinks to a linear spectrum $\hbar\omega = \varepsilon_{k+q} - \varepsilon_k \approx \hbar v_F \cdot q$, independent of k. Consequently, an **1D electron system violates the one-to-one correspondence between low-energy excitations and electrons in Landau FL!**

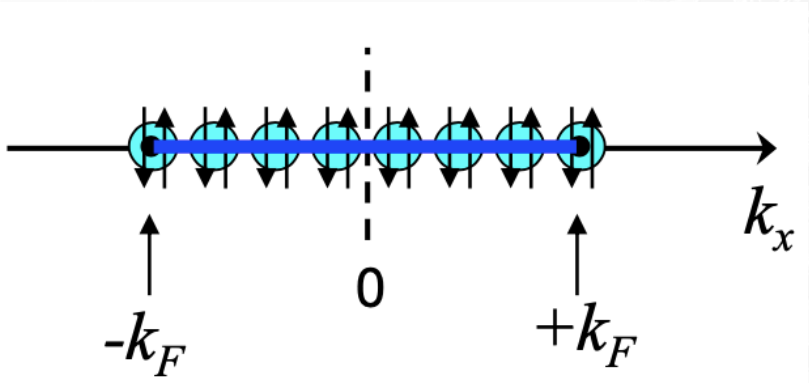


Fig.4. Fermi Surface of 1D system.

This can be explained in real space: **electrons can not avoid each other when they move in 1D** so they have to push each other when they move along a 1D chain. Their motions are highly correlated, even when electron interaction is weak.

Electron-electron interactions are usually very strong in 1D due to inefficient screening. This further enhances the correlations between electrons.

In conclusion, no individual motion/excitation is possible in 1D, the motion is always collective.

02 Part Two

Tomonaga-Luttinger Model: Bosonization and Spin-Charge Separation

Tomonaga Model

Tomonaga model (Tomonaga, 1950) discusses a one-dimensional electron gas, described by Hamiltonian: $H = v_F \sum_{ks} |k| a_{ks}^{\dagger} a_{ks} + \frac{1}{2L} \sum_{k \neq 0} V_k \varrho(k) \varrho(-k)$, where $\varrho(k) = \sum_{p,s} a_{p-k/2,s}^{\dagger} a_{p+k/2,s}$.

The system has length L. Linear dispersion relation holds near Fermi Surface and v_F is the Fermi velocity. $a^{\dagger}$ **and** a **are creation and annihilation operator for electron.** $\varrho(k)$ is the electron density operator. Spin index $s = \pm 1$. V_k is electron-electron interaction term. It will not be $4\pi e^2/k^2$ which is dimensionally incorrect in 1D. **Dimensional analysis suggests the form** $V_k \propto e^2 \left(k_F/k\right)^n$**, where** ***n*** **is any exponent.**

The important physics is the recognition that **the excitations of the electron gas are approximate bosons**, although the elementary particles— electrons— are fermions. **The Tomonaga model assumes that the excitations are exactly bosons, which is the important approximation.**

Bosonization

The basic step in the Tomonaga model is to divide the density operator into two terms:

$\varrho_1(k) = \sum_{s,p>0} a^{\dagger}_{p-k/2,s} a_{p+k/2,s}$ $\varrho_2(k) = \sum_{s,p<0} a^{\dagger}_{p-k/2,s} a_{p+k/2,s}$, then $\varrho(k) = \varrho_1(k) + \varrho_2(k)$.

Tomonaga assumes that these density operators obey the exact commutation relations:

$$\left[\varrho_1(k), \varrho_1\left(-k'\right)\right] = \delta_{kk'}\frac{Lk}{\pi} \ , \ \left[\varrho_2(k), \varrho_2\left(-k'\right)\right] = -\delta_{kk'}\frac{Lk}{\pi} \text{ and } \left[\varrho_1(k), \varrho_2\left(-k'\right)\right] = 0.$$

The commutation relations are not exact, since the commutators give operators. However, **when we take the expectation value of the exact commutation relations in the ground state of the free-particle system, we do get these results.**

Then density operators are expressed in terms of **creation and destruction operators for bosons**:

$$\varrho_1(k) = b_k\left(kL/\pi\right)^{1/2} \ \varrho_1(-k) = b^{\dagger}_k\left(kL/\pi\right)^{1/2} \ \varrho_2(k) = b^{\dagger}_{-k}\left(kL/\pi\right)^{1/2} \ \varrho_2(-k) = b_{-k}\left(kL/\pi\right)^{1/2},$$

where k is always positive and $\left[b_k, b^{\dagger}_{k'}\right] = \delta_{kk'}$.

Tomonaga-Luttinger Model

Bosonization

The electron-electron interaction term will be recast into an interaction between the boson excitations: $\frac{1}{2L}\sum_k V_k \varrho(k)\varrho(-k) = \sum_k \bar{V}_k \left(b_k + b_{-k}^\dagger\right)\left(b_k^\dagger + b_{-k}\right)$, where $\bar{V}_k = \frac{|k|V_k}{2\pi}$.

It's not immediately obvious how to express the kinetic term in terms of the new boson operators. When faced with this predicament, we instead **examine the approximate commutation relations**: $\left[b_k, H_0\right] = k v_F b_k \equiv \omega_k b_k$, where $H_0 = v_F \sum_{ks} |k| a_{ks}^\dagger a_{ks}$ and $\omega_k = v_F |k|$.

The one-dimensional electron gas has now been recast into the **boson Hamiltonian with several key approximations on commutation relations**:$H = \sum_k \omega_k b_k^\dagger b_k + \sum_k \bar{V}_k \left(b_k + b_{-k}^\dagger\right)\left(b_k^\dagger + b_{-k}\right)$.

This form is a description of the electron gas as due to boson excitations and is exactly solvable.

Tomonaga-Luttinger Model

Excitation Spectrum of Density Oscillations

The Hamiltonian $H = \sum_k E_k \left(\alpha_k^\dagger \alpha_k + \frac{1}{2} \right)$ can be constructed by changing operators:

$b_k + b_{-k}^\dagger = \left(\omega_k / E_k\right)^{1/2} \left(\alpha_k + \alpha_{-k}^\dagger \right) \& \; b_k^\dagger - b_{-k} = \left(E_k / \omega_k\right)^{1/2} \left(\alpha_k^+ - \alpha_{-k} \right)$, where $E_k = \sqrt{\left(\omega_k^2 + 4\omega_k \bar{V}_k\right)}$.

So far the form of the interaction potential V_k hasn't been specified. In fact, **physicists often choose a variety of forms for this interaction in order to suit their problem.** In the electron gas, there are two different types of excitations.

One excitation is described by $V_k = (2/3)\left(e^2 k_F^2 / k^2\right)$. This leads to the **plasma modes at long wavelength with plasma frequency** $\omega_p (= 4\pi e^2 n / m$, here $n = k_F^3 / 3\pi^2)$: $E_k = \sqrt{\left(k^2 v_F^2 + \omega_p^2\right)}$.

Another is **the electron-hole excitations at short wavelength, which is probably best described by the choice** $V_k \propto e^2 = V_0 \quad (V_0 > 0)$**.** The energy spectrum is just altered by having the Fermi velocity increased.

Tomonaga-Luttinger Model

Spin-Charge Separation

Overhauser (1965) has shown that **the excitation spectrum is completely described by the sum of these two types of excitations: density oscillations and spin waves.** The spin waves are described by the operators: $\sigma(k) = \sigma_1(k) + \sigma_2(k)$ $\sigma_1(k) = \sum_{s,p>0} s a^{\dagger}_{p-k/2,s} a_{p+k/2,s}$ $\sigma_2(k) = \sum_{s,p<0} s a^{\dagger}_{p-k/2,s} a_{p+k/2,s}$,

where spin index $s = \pm 1$ represents $\uparrow, \downarrow$.

Define $\varrho_s = \sum_p a^{\dagger}_{p-k/2,s} a_{p+k/2,s}$, then we can see that $\varrho = \varrho_{\uparrow} + \varrho_{\downarrow}$ and $\sigma = \varrho_{\uparrow} - \varrho_{\downarrow}$.

In the same spirit which was used to derive the approximate commutation relations for the density operators, we deduce a similar approximate set of commutators: $\left[\sigma_i(k), \sigma_j\left(-k'\right)\right] = \delta_{ij}\delta_{kk'} kL/\pi$

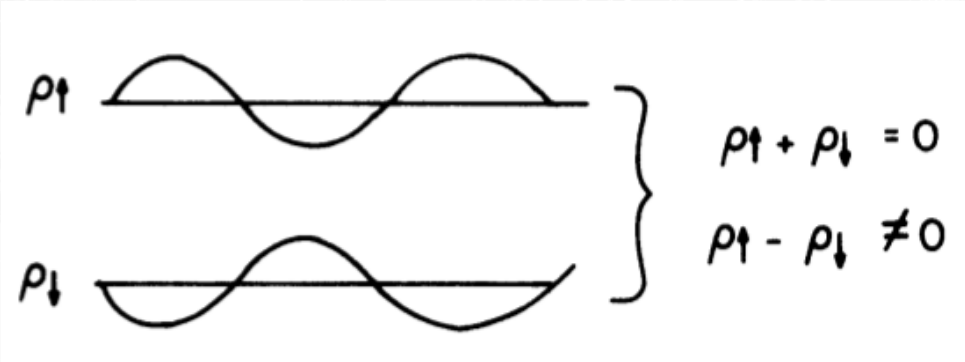


Fig.5. Illustration of spin up and spin down charge densities.

The spin operators commute with the density operators when the two spin states are occupied with equal probability— so that the system is not magnetic, and so describe an independent set of excitations, otherwise the excitation spectrum is quite different.

These excitations can be represented by a new set of creation and destruction operators, which for $k > 0$ are:$\sigma_1(k) = c_k\sqrt{kL/\pi}$ $\sigma_1(-k) = c_k^\dagger\sqrt{kL/\pi}$ $\sigma_2(k) = c_{-k}^\dagger\sqrt{kL/\pi}$ $\sigma_2(-k) = c_{-k}\sqrt{kL/\pi}$, where $[c_k, c_{k'}^\dagger] = \delta_{kk'}$ and $[c_k, b_{k'}^\dagger] = [c_k, b_{k'}] = [c_k^\dagger, b_{k'}^\dagger] = 0$.

The commutator with the kinetic energy term can be evaluated in the same approximation, thus the spin wave part of the Hamiltonian is: $H_{sw} = \sum_k [\sigma_1(-k)\sigma_1(k) + \sigma_2(k)\sigma_2(-k)] = \sum_k \omega_k c_k^\dagger c_k$.

The total Hamiltonian H_T for the excitation spectra of the one-dimensional electron gas then is: $H_T = \sum_k (E_k\alpha_k^\dagger\alpha_k + \omega_k c_k^\dagger c_k)$, having the density and spin wave excitations decoupled.

Tomonaga-Luttinger Model

Luttinger Model

A model proposed by Luttinger (1963) is a slight variation on the Tomonaga model. It has the advantage of being exactly solvable, with fewer approximations.

The basic feature of the Luttinger model is that the system has two types of fermions. One has an energy spectrum given by $\varepsilon_k = k v_F$, while the other has an energy spectrum given by $\varepsilon_k = -k v_F$. They are shown by the solid and dashed lines in Fig.6(a). **He made an assumption that an infinite number of each kind of particle, since the occupied energy states stretch to negative infinity.**

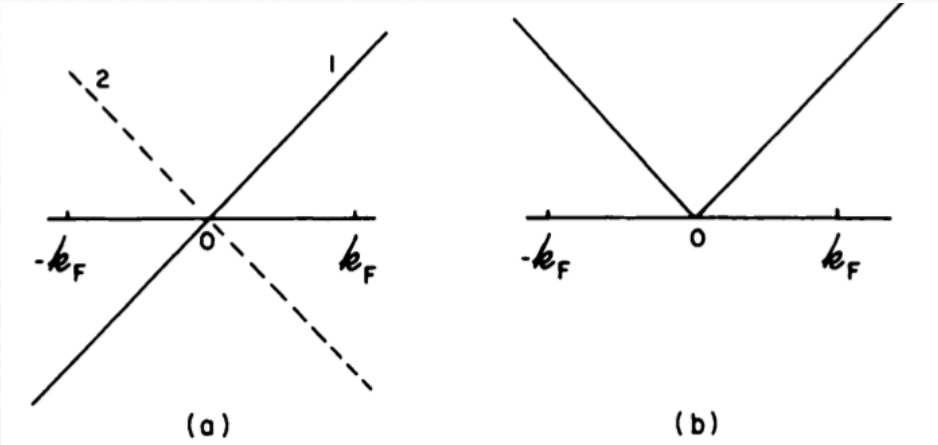


Fig.6. (a).The Luttinger model has two distinct particles, with separate energy bands. (b). The Tomonaga model has one particle, whose energy band is $|k| v_F$.

Luttinger Model

The two kinds of fermions in the Luttinger model are denoted by the operators $a_{1,k,s}$ and $a_{2,k,s}$, where the subscript 1 or 2 designates the particle. **The two bands are quite independent, so the two fermion operators anti-commute.** Operators $\rho_i(k)$ and $\sigma_i(k)$ are defined in a manner analogous to the Tomonaga model ($p > 0$): $\varrho_i(p) = \sum_{k,s=\pm 1} a^{\dagger}_{i,p+k,s} a_{i,k,s}$ $\sigma_i(p) = \sum_{k,s=\pm 1} s a^{\dagger}_{i,p+k,s} a_{i,k,s}$

$\left[\sigma_i(-p), \sigma_j\left(p'\right)\right] = \delta_{ij}\delta_{pp'} pL/\pi$ $\left[\varrho_i(-p), \varrho_j\left(p'\right)\right] = \delta_{ij}\delta_{pp'} pL/\pi$ $\left[\sigma_i(p), \varrho_j\left(p'\right)\right] = 0$. **These commutation relations depend, in an important way, on the assumption that there is an infinite number of negative energy particles.**

Kinetic energy term in the Luttinger model $H_0 = v_F \sum_{s,k} (a^{\dagger}_{1,k,s} a_{1,k,s} - a^{\dagger}_{2,k,s} a_{2,k,s})$ can be exactly represented by: $H_0 = v_F \pi/L \sum_{s,k} (\varrho_1(p)\varrho_1(-p) + \varrho_2(-p)\varrho_2(p) + \sigma_1(p)\sigma_1(-p) + \sigma_2(-p)\sigma_2(p))$

Luttinger Model

The transformation to boson operators is: $\varrho_1(-p) = b_{1,p}\left(pL/\pi\right)^{1/2}$ $\varrho_2(-p) = b^{\dagger}_{2,-p}\left(pL/\pi\right)^{1/2}$

$\sigma_1(-p) = c_{1,p}\left(pL/\pi\right)^{1/2}$ $\sigma_2(-p) = c^{\dagger}_{2,-p}\left(pL/\pi\right)^{1/2}$. **For particle 2, the "Fermi surface" is at** $-k_F$. Electron-hole pairs are made mostly at negative wave vectors. Thus the operator $a^{\dagger}_{2,k,s}a_{2,k+p,s}$ for $p > 0$ & $k < -k_F$ takes an electron from the occupied state $-k_F < k + p$ to the unoccupied state $k < -k_F$. The summation over all such electron-hole pairs is represented by boson creation operator $b^{\dagger}_{2,-p}$.

The Luttinger model has the advantage of being exactly solvable: various kinds of interaction terms may be added to the Luttinger model, provided the total fermion Hamiltonian's charge and spin excitations are gapless. **The disadvantage of the model is that it is unphysical, since it contains the infinite reservoir of negative energy particles.**

Tomonaga-Luttinger Model

Luttinger Liquid

A Luttinger liquid is a paramagnetic 1D metal without Landau quasi-particle excitations.

"Paramagnetic" and "metal" require that **the spin and charge excitations are gapless**, more precisely with dispersions $\omega_\nu \approx v_\nu |q|$ ($\nu = \rho, \sigma$ for charge and spin). The charge and spin modes (holons and spinons) possess different excitation energies ($v_\rho \neq v_\sigma$) and are bosons. This leads to the **separation of charge and spin of an electron (or hole)** added to the Fermi sea.

The bosonic nature of charge and spin excitations, together with the reduced dimensionality leads to a peculiar kind of short-range order at T = 0. The system is at a (quantum) critical point, with power-law correlations, and **the scaling relations between the exponents of its correlation functions are parameterized by renormalized coupling constants K_ν. The individual exponents are non-universal, i.e. depend on the interactions.** For Luttinger liquids, K_ν is the equivalent of the Landau parameters f.

Predictions for Experiments

The thermodynamics is not qualitatively different from a Fermi liquid, with a linear-in-T specific heat, and T-independent Pauli susceptibility and electronic compressibility:

$C_v = (v_F/v_\rho + v_F/v_\sigma)\gamma_0/2 \cdot T,\ \chi = 2K_\sigma/\pi v_\sigma,\ \kappa = 2K_\rho/\pi v_\rho.$

The electronic structure factor $S(k)$ and NMR spin-lattice relaxation rate T_1^{-1}, indicating charge and spin correlation respectively, translate into: $S(k) \sim |k - 2k_F|^{K_\rho} + |k - 4k_F|^{4K_\rho - 1}$, $T_1^{-1} \sim T + T^{K\rho}$. **The structure factor can be interpreted as showing fluctuations both of Peierls-type** $(2k_F)$ **and of Wigner-crystal-type** $(4k_F)$ **charge density waves, and the two terms in** T_1^{-1} **come from the** $q \approx 0$ **and** $2k_F$ **spin fluctuations.**

If we consider electron-electron scattering in a band with filling factor 1/n, we obtain from the current-current correlations: $\rho(T) \sim T^{n^2 K_\rho - 3}$, $\sigma(\omega) \sim \omega^{n^2 K_\rho - 5}$.

03 Part Three

Experiments: Predictions and Verifications

Experiments

Candidate Materials and Experimental Probes

Materials that can be regarded as 1D electron system:

Chain-like compounds (TTF-TCNQ, $SrCuO_2$, …)

Edge state of Fractional Quantum Hall Effects

Mesoscopic quantum wires, both on semiconductor and tube base (Carbon nanotubes/ Gold wires deposited on a Si surface/ …)

Experimental probes that can be used:

PES, especially ARPES (dispersion of spinons and holons, different velocities): Nature Physics 2, 397 - 401 (2006)

Tunneling spectroscopy (power law dependence of DOS on energy): Nature |VOL 397| 18 (1999), PRL 87, 18(2001)

Transport (power law dependence of conductivity on temperature): *Science* **275,** 1922–1925 (1997)

Experiments

Spin-charge Separation in 1D Mott Insulator

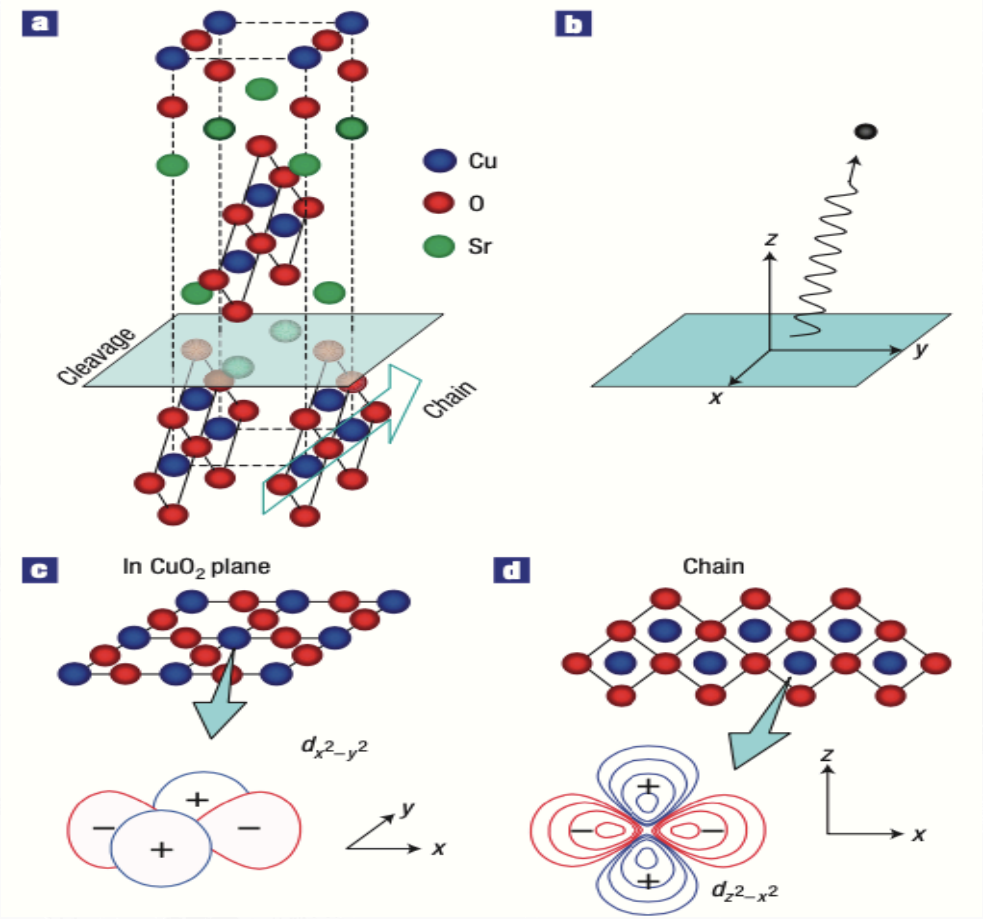


Fig.7. Crystal structure, cleavage plane and relevant orbitals. $SrCuO_2$: quasi-1D chain, AF Mott-insulator ground state

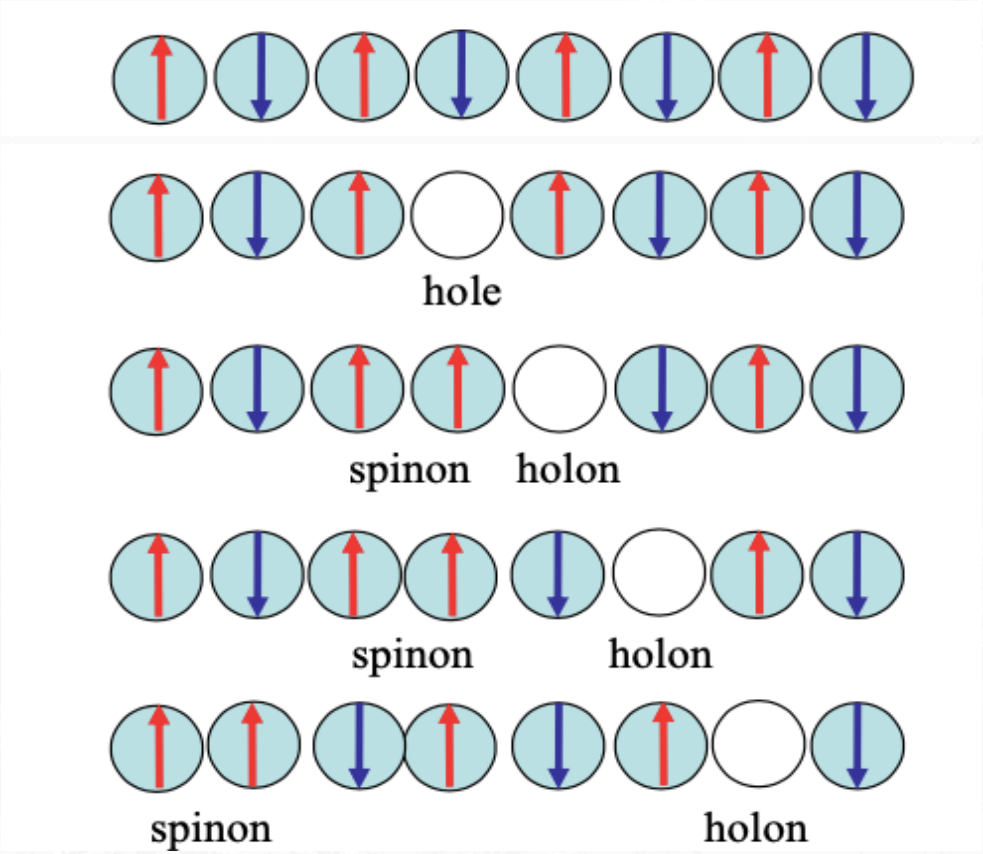


Fig.8. Illustration of Spin-Charge Separation in a 1D chain with one electron per site with strong onsite Coulomb repulsion. In ARPES, photon excites a hole.

Experiments

Spin-charge Separation in 1D Mott Insulator

In LL, for each k the single qp peak splits into a spinon–holon two-peak structure, due to the different velocity of spinon and holon. **Both peaks disperse towards the low-binding-energy side as the momentum increases, with the high-energy feature dispersing faster than the low-energy feature.** The holon (lower branch) disperses faster (with larger velocity) than the spinons (upper branch).

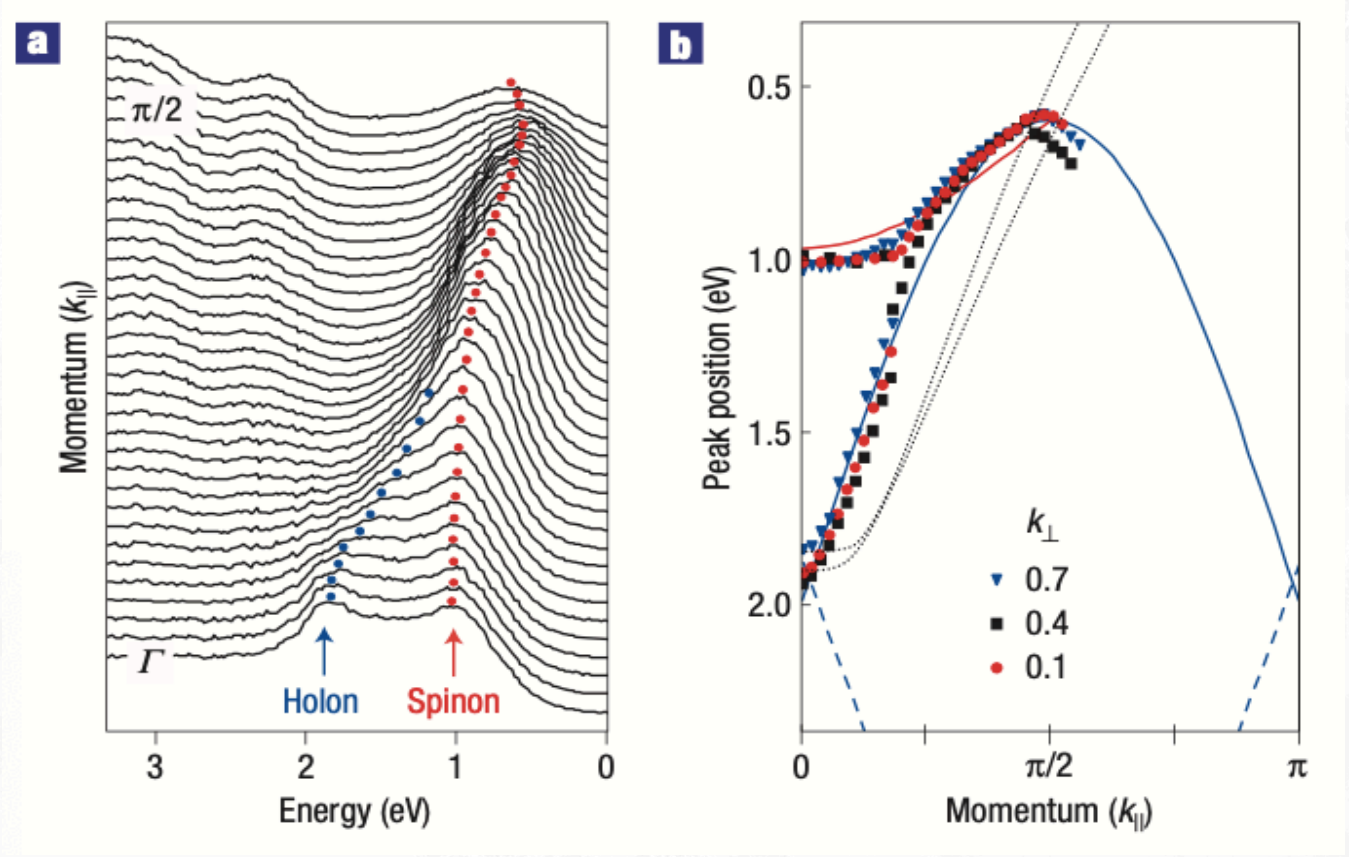
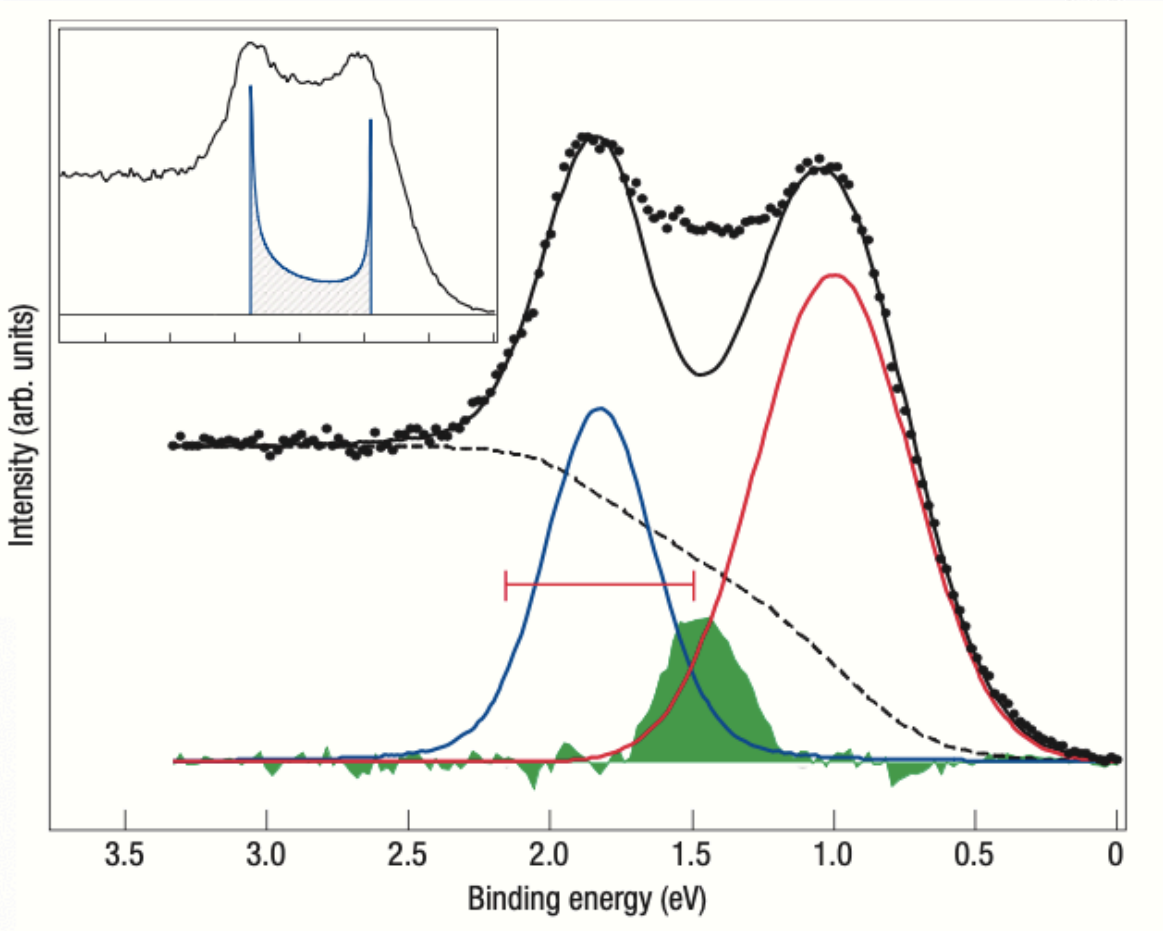


Fig.9.(a). energy distribution curves for different $k_{||}$ values showing two-peak feature. (b). experimental (symbols) and theoretical (solid and dashed lines) dispersions.

Experiments

Spin-charge Separation in 1D Mott Insulator

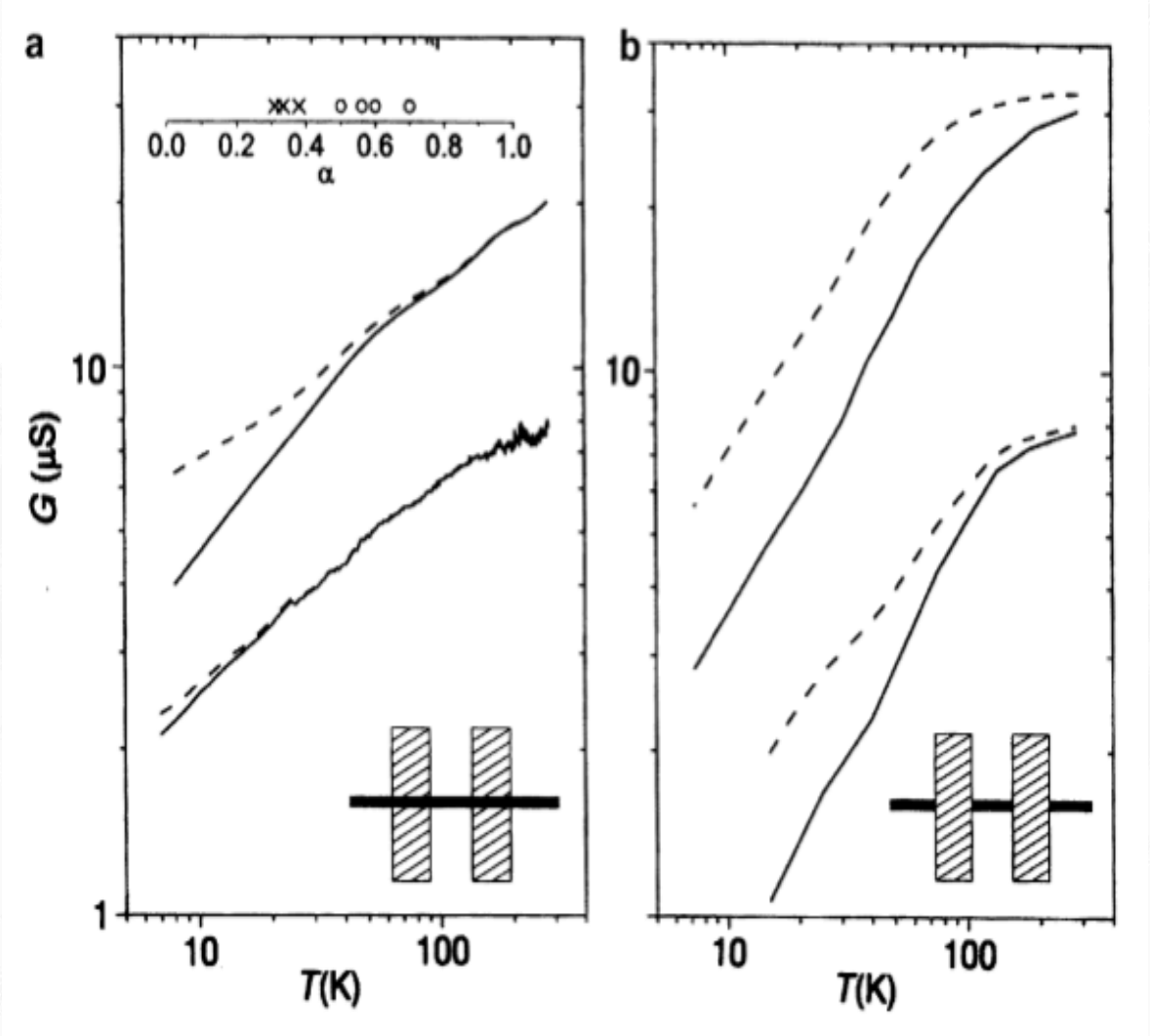
Fig.10. Spectral function of $k_{||} = \Gamma$ *along the chain direction. The raw data (black dots) show apparent two-peak structure. The solid black line is the sum of two Gaussian peaks (red line: spinon, blue: holon) and an integrated background (dashed line). The inset shows the expected underlying spectral function with edge singularities, obtained from the Beth–Ansatz solution of the Hubbard model.* ***The extra weight is consistent with an underlying spectral function with the double-edge singularities that are expected in the spin–charge separation picture.***



Experiments

Transport Measurements in Carbon Nanotubes

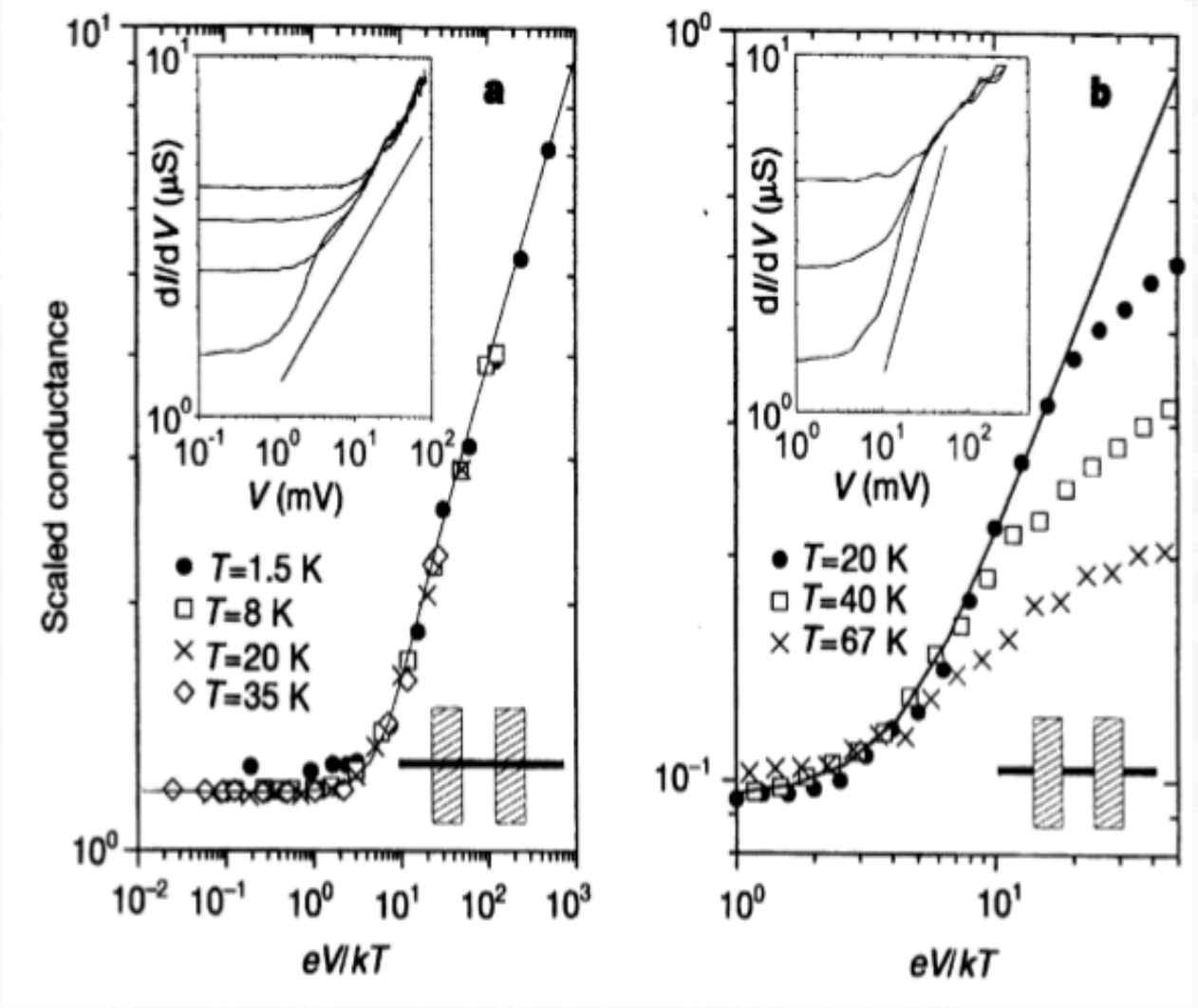
*Fig.11. Conductance plotted against temperature for individual nanotube ropes. (**a**). Data for ropes that are deposited over pre-defined leads (bulk-contacted); (**b**). data for ropes that are contacted by evaporating the leads on top of the ropes (end-contacted). Sketches depicting the measurement configuration are shown in the lower insets. The plots show both the raw data (solid line) and the data corrected for the temperature dependence expected from the Coulomb blockade (CB) model (dashed line). The upper inset to (**a**) shows the power- law exponents inferred for a variety of samples.*



Experiments

Tunneling Measurements in Carbon Nanotubes

*Fig.12. The differential conductance dI/dV(proportional to G(T) in linear response) measured at various temperatures. Inset in (**a**) and (**b**) are dI/dV curves taken on a bulk-contacted rope and an end-contacted rope respectively for different temperatures. In both insets, a straight line on the two log plot is shown as a guide to the eye to indicate power-law behaviour. The main panels (**a**) and (**b**) show these measurements collapsed onto a single curve by using the scaling relations. The solid line is the theoretical result fitted.*



04 Part Four

Further Discussions: Stability and Open Questions

Further Discussions

Stability of Luttinger Liquid

Luttinger Liquid theory crucially relies on one-dimensionality. Moreover, most of our discussion was for T = 0, and ignored phonons, lattice effects, impurities, etc. Are these factors detrimental to Luttinger liquids? In many cases, the answer will depend on the scales one considers.

Finite temperature doesn't count, and the correlation functions mentioned above can be calculated for T > 0. However, charge-spin separation will be masked in the spectral function when $(v_\rho - v_\sigma)q < T$.

Interchain tunneling will introduce 3D effects. Depending on the on-chain interactions, it will either produce a crossover to a Fermi liquid (weak interactions), or to a long-range ordered 3D insulating or superconducting phase (strong interactions). **In any event, a Luttinger liquid is unstable towards 3D coupling at low enough temperature (scales).**

Further Discussions

Stability of Luttinger Liquid

Various studies of phonons coupled to Luttinger liquids have shown that depending on details of the electron-electron and electron-phonon interactions, **a Luttinger liquid may remain stable, though renormalized, when phonons are added. Alternatively, the electron-phonon interaction could lead to the opening of a spin gap, and thereby destabilize the Luttinger liquid. But the correlation functions continue to carry certain remnants of Luttinger physics**, like non-universal power laws (the system remains conducting), and charge-spin separation.

When the crystal lattice is important (commensurate band filling), the system may become insulating. For a 1D band insulator, Luttinger liquid physics is expected to be lost completely, although not much is known firmly. **More interesting is the case of a Mott insulator, brought about by electronic correlations. However even here, charge-spin separation is still seen in experiments.** Moreover, far above the (charge or spin) gaps, they should no longer influence the physics, and genuine Luttinger liquid behavior is expected there.

Further Discussions

Aspects of Mesoscopic Systems

Due to the small sample size, boundary conditions become of importance, and may dominate the physics.

The influence of isolated impurities on transport, or tunneling through quantum point contacts, is an important problem. **The physical origin of this effect is the establishment of a strong Friedel oscillation around the impurity which will increasingly backscatter the electrons at lower energy scales.**

An impurity can therefore be assimilated with open boundary conditions. Quite generally, 1D interacting fermions with open boundaries and gapless excitations form a bounded Luttinger liquid state, rather similar to ordinary Luttinger liquids but with a different set of exponents and scaling relations. **The K_v are properties of the Hamiltonian, and therefore independent of boundary conditions.**

Further Discussions

Open Questions

Many important questions remain open, both in theory and experiment.

One important problem relates to scales. Can both power laws and charge-spin separation be observed over the entire energy range? Do they depend on the specific Hamiltonian considered, e.g. on the interaction strength and range, and how?

Concerning mesoscopic systems, only Luttinger liquids with open boundaries are thoroughly characterized. It is conceivable that **other boundary conditions** (Fermi liquid leads, boundary fields or spins, superconductors) lead to new sets of critical exponents.

What is the spectral weight associated with Luttinger liquid physics in any given microscopic model, or in any given experimental system? How sure can we be that the weight of the coherent spin and charge modes in the Green's function is sufficiently high, so that experiments (e.g. photoemission) actually see these excitations, and not just incoherent contributions or bare high-energy excitations?


Thanks for Attention!

05 Part Five

Appendix

Appendix

Theory of Break-down of Landau FL

Fermi Liquid theory breaks down for one-dimensional (1D) metals. Technically, this happens because **some vertices Fermi liquid theory assumes finite (those involving a $2k_F$ momentum transfer) actually diverge because of the Peierls effect.** An equivalent intuitive argument is that in 1D, perturbation theory never can work even for arbitrarily small but finite interactions: when **degenerate perturbation theory is applied to the coupling of the all-important electron states at the Fermi points $\pm k_F$, it will split them and therefore remove the entire Fermi surface!** Free-electron-like metal will therefore not be stable in 1D. The underlying physical picture is that the coupling of quasi-particles to collective excitations is small in 3D but large in 1D, no matter how small the interaction.

Appendix

Bosonization

To complete the bosonization program, a local fermion operator must be expressed in terms of bosons. Exact operator identities are available for the Luttinger model, which can be summarized schematically as :

$$\Psi_s(x) \sim \exp\left\{ i \sum_{\nu} \sum_{p} e^{ipx} (\ldots) \left(b_{\nu,p} + b^{\dagger}_{\nu,-p} \right) \right\}$$

This fermion-boson transformation turns bosonization into a useful device: **all correlation functions can be calculated as simple harmonic oscillator averages.** As a consequence, Luttinger liquid predictions for all physical properties can be produced.

Bosonization is an easy and transparent way to calculate the properties of Luttinger liquids. However, it is not the only method. More general, and more powerful is the direct application of conformal field theory to a microscopic model of interacting fermions. For Luttinger liquids, both methods become identical, and one might view bosonization as solid state physicist's way of doing conformal field theory. Also Green's functions methods have been used successfully.

Appendix

Roles of Electron-electron Interaction

One possibility is that the **interactions open a gap in the spin and/or charge excitation spectrum.** The system then no longer is paramagnetic and/or metallic. **With a charge gap, we have a 1D Mott insulator, with a spin gap a conducting system with strong charge density wave or superconducting correlations, and gaps in both channels imply a band insulator.** Luttinger liquid theory cannot be applied anymore.

In the other case, charge and spin excitations remain gapless: a Luttinger liquid is formed. Then, electron-electron interactions will make $v_\rho \neq v_\sigma \neq v_F$, leading to charge-spin separation.

Interactions will also renormalize the electronic compressibility and magnetic susceptibility, and the charge and spin stiffnesses, and by comparing the velocities measuring this renormalization to v_ν, the correlation exponents K_ν can be defined. The K_ν therefore only depend on the low-energy properties of the Hamiltonian. **Two parameters per degree of freedom, K_ν and v_ν, completely describe the physics of a Luttinger liquid.**

Appendix

Cross-section Effect

The spectral weight of the main valence band near the Γ point is strongly suppressed. In the low-photon-energy data, the spectral weight of the main valence band is large (at least a few times the Zhang–Rice singlet feature). In that case, the holon edge is completely buried under the tail of the main valence band.

In addition, the two-peak structure is prominent only for a certain region of $k_\perp$ (high-$k_\perp$ values). High-energy photons worked favourably in suppressing the main valence band spectral weight, which in combination with data over a large region in the momentum space allowed us to observe distinct spinon and holon branches.

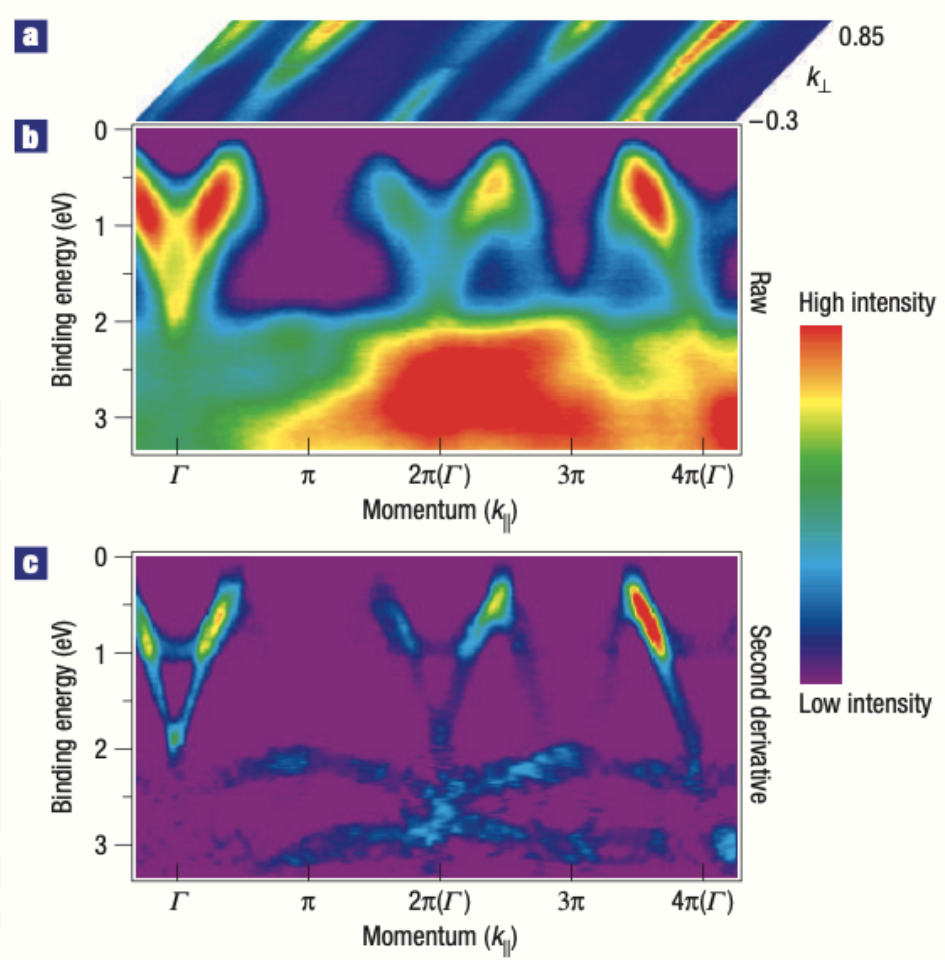


Fig.13. Colour-scale plot of ARPES data.

Appendix

Structure of Carbon Nanotubes

SWNTs are deposited from a suspension in dichloroethane onto a 1-mm-thick layer of SiO_2 that has been thermally grown on a degenerately doped Si wafer, used as a gate electrode. Atomic force microscopy imaging reveals that the diameters of the ropes vary between 1 and 10 nm.

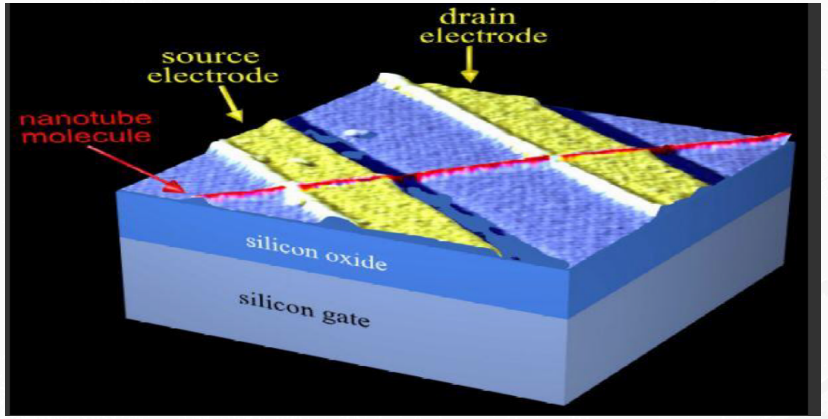


Fig.14. FET of SWCNT bundle, with both semiconducting and metallic tubes in it, but transport is dominated by metallic ones.

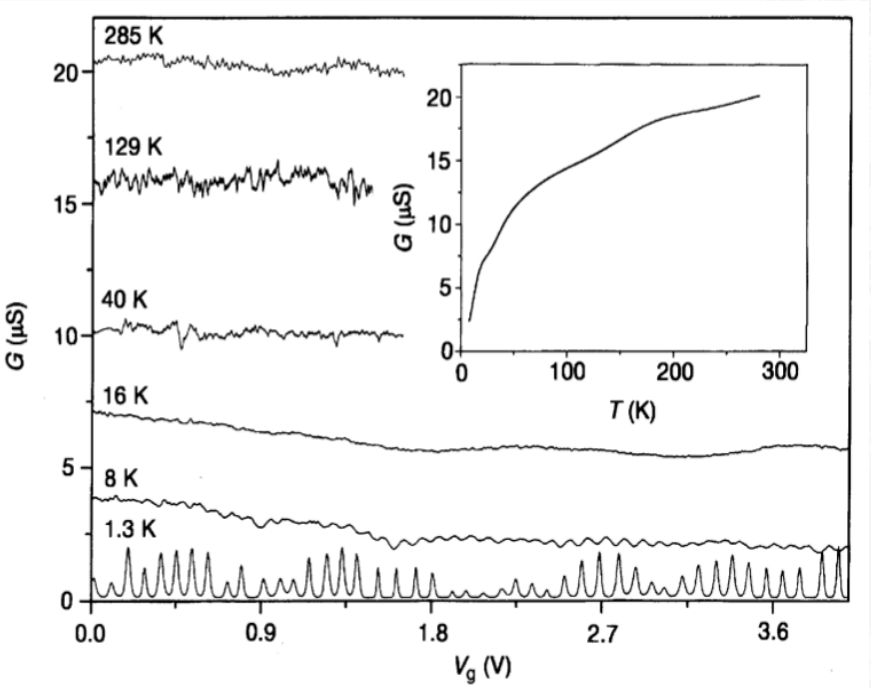
In the first method, from measurements in the Coulomb blockade regime, we learn that the electrons are confined to the length of rope between the leads. This implies that the leads cut the nanotubes into segments, and transport involves tunneling into the ends of the nanotubes (`end-contacted').

In other, samples were selected that showed Coulomb blockade behaviour at low temperatures with a single well-defined period, indicating the presence of a single quantum dot. The charging energy of these samples indicates a quantum dot with a size substantially larger than the spacing between the leads. Transport thus occurs by electrons tunneling into the middle, or bulk, of the nanotubes (`bulk-contacted').

Appendix

Other Possible Explanations for the Data

A possible explanation for the approximate power-law behaviour is a strong energy dependence of the tunnel barrier, with increased tunnelling efficiency at high energies. This would lead to activated transport over the barrier, so that the conductance can be described by ,$G \sim exp(-V_b/kT)$, where V_b is the height of the tunnel barriers. However, this is inconsistent with the fact that the conductance extrapolates to G~0 at T~0.



The type of power-law behaviour could also arise if the electron transport were to occur through multiple quantum dots in series, formed by disorder or by barriers produced when the nanotubes bend over the lithographically defined contacts. But as we have chosen to study only nanotube ropes that exhibit a single dominant period for the Coulomb oscillations at low temperatures, our samples are likely to contain only a single quantum dot.