固体物理第十三次作业

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$$\begin{aligned}\delta F &= \int \mathrm{d}^3\boldsymbol{r}\left[-\boldsymbol{j}\cdot\delta\boldsymbol{A}+\frac{1}{\mu_0}\boldsymbol{B}\cdot(\nabla\times\delta\boldsymbol{A})\right]\\ &= \int \mathrm{d}^3\boldsymbol{r}\left[-\boldsymbol{j}\cdot\delta\boldsymbol{A}-\frac{1}{\mu_0}\nabla\cdot(\boldsymbol{B}\times\delta\boldsymbol{A})+\frac{1}{\mu_0}(\nabla\times\boldsymbol{B})\cdot\delta\boldsymbol{A}\right]\\ &= \int \mathrm{d}^3\boldsymbol{r}\left[-\boldsymbol{j}\cdot\delta\boldsymbol{A}+\frac{1}{\mu_0}(\nabla\times\boldsymbol{B})\cdot\delta\boldsymbol{A}\right]\end{aligned}$$

$$\frac{\delta F}{\delta\boldsymbol{A}(\boldsymbol{r})}=-\boldsymbol{j}+\frac{1}{\mu_0}\nabla\times\boldsymbol{B}=0$$

$$\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}$$

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显然

$$|\psi_0|^2=-\frac{a}{b},\quad \frac{1}{2}\mu_0H_c^2-\frac{a^2}{2b}=\frac{B_c^2}{2\mu_0}-\frac{a^2}{2b}=0$$

对自由能变分得

$$\frac{\delta F}{\delta\psi^*(x)}=\frac{\delta F_{\mathrm{matter}}}{\delta\psi^*(x)}=\frac{(\boldsymbol{p}+e^*\boldsymbol{A})^2}{2m^*}\psi(x)+a\psi(x)+b|\psi(x)|^2\psi(x)=0$$

$$\Rightarrow\int_{-\infty}^{\infty}\mathrm{d}x\left[\frac{1}{2m^*}|(\boldsymbol{p}+e^*\boldsymbol{A})\psi(x)|^2+a|\psi(x)|^2+b|\psi(x)|^4\right]=0$$

界面系统的吉布斯自由能为

$$\begin{aligned}G_s &= F-\mu_0\int\mathrm{d}^3\boldsymbol{r}\boldsymbol{H}\cdot\boldsymbol{M}\\ &= \int\mathrm{d}^3\boldsymbol{r}f_{\mathrm{matter}}+\int\mathrm{d}^3\boldsymbol{r}\left(\frac{\boldsymbol{B}^2}{2\mu_0}-\mu_0\boldsymbol{H}\cdot\boldsymbol{M}\right)=\int\mathrm{d}^3\boldsymbol{r}f_{\mathrm{matter}}+\frac{\mu_0}{2}\int\mathrm{d}^3\boldsymbol{r}(\boldsymbol{H}^2+\boldsymbol{M}^2)\\ &= \int\mathrm{d}y\mathrm{d}z\int_{-\infty}^{\infty}\mathrm{d}x\left\{\left[\frac{1}{2m^*}|(\boldsymbol{p}+e^*\boldsymbol{A})\psi(x)|^2+a|\psi(x)|^2+\frac{b}{2}|\psi(x)|^4\right]+\frac{\mu_0}{2}[H_c^2+M(x)^2]\right\}\\ &= \int\mathrm{d}y\mathrm{d}z\int_{-\infty}^{\infty}\mathrm{d}x\left\{-\frac{b}{2}|\psi(x)|^4+\frac{\mu_0}{2}[H_c^2+M(x)^2]\right\}\\ &= \frac{\mu_0}{2}H_c^2\int_{-\infty}^{\infty}\mathrm{d}x\left[-\left|\frac{\psi(x)}{\psi_0}\right|^4+1+\frac{M(x)^2}{H_c^2}\right]\int\mathrm{d}y\mathrm{d}z\end{aligned}$$

正常态$f_{\mathrm{matter}}=0$, $\boldsymbol{M}=0$, 吉布斯自由能为

$$G_n=\int\mathrm{d}^3\boldsymbol{r}f_{\mathrm{matter}}+\frac{\mu_0}{2}\int\mathrm{d}^3\boldsymbol{r}(\boldsymbol{H}^2+\boldsymbol{M}^2)=\frac{\mu_0}{2}H_c^2\int_{-\infty}^{\infty}\mathrm{d}x\int\mathrm{d}y\mathrm{d}z$$

故

$$G_s-G_n=\frac{\mu_0}{2}H_c^2\int_{-\infty}^{\infty}\mathrm{d}x\left[-\left|\frac{\psi(x)}{\psi_0}\right|^4+\frac{M(x)^2}{H_c^2}\right]\int\mathrm{d}y\mathrm{d}z=\sigma\int\mathrm{d}y\mathrm{d}z$$

$$\Rightarrow\sigma=\frac{\mu_0}{2}H_c^2\int_{-\infty}^{\infty}\mathrm{d}x\left[-\left|\frac{\psi(x)}{\psi_0}\right|^4+\frac{M(x)^2}{H_c^2}\right]=\frac{\mu_0}{2}H_c^2\int_{-\infty}^{\infty}\mathrm{d}x\left[\left(\frac{B(x)}{B_c}-1\right)^2-\left|\frac{\psi(x)}{\psi_0}\right|^4\right]$$